Chapter Three

§3.1. *Synopsis: Chapter Three.*

The logarithm of 10 is taken conveniently as 1,00000,00000,0000. The main sequence of numbers in continued proportions is defined to be the powers of ten, in which the individual powers or indices correspond to the rational logarithms 1, 2, 3,... etc. Two other sequences of numbers in continued proportion are presented with fractional rational logarithms, based on a common difference of the logarithms of 0.25 and $0.1\dot{3}$. The prime numbers will have logarithms which are irrational numbers; however, these will be approximated by rational numbers to the precision required, which is equivalent to 14 decimal places.

§3.2. *Rational Logarithms.*

With the Logarithm of Unity settled, in order that we may look for another number, it is the nearest one which will be the most frequently used and certainly the most necessary, and to that number we may assign some convenient logarithm, which shall be both easily remembered and copied out as often as necessary. Now from all the numbers, none seem to be more outstanding or adapted to this task than ten, of which the logarithm shall be 1,00000,00000,0000[1].

Therefore let the particular numbers be one and ten, and the logarithms of these 0 and 1,00000,00000,0000. Moreover we have set up these four numbers first, neither persuaded by some need, but by choice; nor by considering the truth of the workings of the Arithmeticians[2] (which can be established in many different ways of the most diverse kinds), but ease of use. The remaining logarithms (since for these to be established, there will not be a single whole number to discuss by choice) are thus to be prepared for all these primes[3] in order that, with the greatest agreement all the way from beginning to end, they will comply with the same rules, and will be found to give faithful service in their use.

While some of these logarithms which we seek are rational numbers, which painstakingly will be found and shown, others are irrational numbers, each one of which is incomplete, but we will give the nearest correct complete or rational number, in order that the numerical values of these numbers will differ by less than one from their true value,[*i.e.* in their final digit][4].

Rational logarithms are to be set apart not only for those two numbers one and ten, with which the structure of all the remaining logarithms rests from the foundations, as it were - but for all numbers which can be assigned with the same two in whatever way to the same series of continued proportions, as we see here [in Table 3-1.] :

A	*B*	*A*	*C*	*B*
1	0	1	1	0
10	100000	__. 10 - -- -- -	177827941	025000
100	200000	_. 10	316227766	050000
1000	300000	__.1000 - - - - -	562341325	075000
10000	400000	10	10	100000
		__.100000 - -	177827941	125000
		_.1000	316227766	150000
		__.10000000	562341325	175000
		100 - - - -	100 - - - -	200000

A	*C*	*B*
1	1	0
_.(6) 10	146779926768	$0166\frac{2}{3}$
_.(3) 10	215443469003	$0333\frac{2}{3}$
_.(2) 10	3162776017	0500
_.(3) 100	464158883360	$0666\frac{2}{3}$
_.(6) 100000	681292069054	$083\frac{1}{3}$
10	10 - - - - - -	10000

[Table 3-1]

A are numbers in continued proportion from unity; *C* are the absolute values of the same [*i.e.* to the specified number of places][5] , which are also continued proportions; column *B* are the correct rational logarithms of these.

All the remaining numbers, which do not fall into any of these kinds of continued proportional series described (into which the two given numbers one and ten have been placed), have irrational logarithms, which cannot be accurately expressed either in whole numbers or in the parts we call fractions. But although we cannot obtain these irrational logarithms accurately, nevertheless we shall be able to find precisely these rational logarithms nearest the correct values, for if we examine the use between the irrational and the rational logarithms, there will be no difference between them. From this same malady of Ptolemy[*i.e.* the occurrence of rounded irrational quantities], the tables of the subtended chords, and of all tables subsequent to these up to the time of the astronomers' tables of sines, tangents, and secants, have suffered without any more serious disadvantage.

§3.3. *Notes On Chapter Three.*

1 As indicated in the last chapter, the choice of 0 for the log 1 and now 1,00000,00000,0000 for log 10 do not follow from any reason more pressing than the desire to split the interval between 0 and 1 into a large dense set of 1,00000,00000,0000 proportional numbers, with ease of use in performing accurate calculations justifying this particular choice.

2 Perhaps Briggs is thinking here about Napier's tables, where allowances are continually being made for the factor w, or 10^7, the number in Napier's scheme for which log w = 0 ; also, Briggs and Napier had together considered several schemes, during Briggs' visits to Merchiston castle in his vacations; or of the other sets of tables that had appeared, such as those of Kepler (for 1000 integers only) or the more extensive tables of Ursus or Burgi. In his own case, these extra complications do not exist. Indeed, Briggs' tables went on to be the preferred ones for the following 350 years, promulgated by Vlacq.

3 Briggs evaluates the logs of 2 and 7 in Chapter 5 by a completely different method (due to Napier) from the main approach he is to adopt in chapter 6, thus providing some sense of confidence in his calculations. Initially, he concentrates on numbers which have rational logs.

4 Briggs is pointing out that irrational logarithms do not have a finite decimal expansion: however, to the large number of significant places he is working to, (usually 17) there will be little difference between the irrational log and that bounded above and below by rational logs which give less than one in their variation of the final significant figure. He habitually worked to several more places than required, and rounded down at the end of the calculation to 14 places. This does not, of course, mean that his tables were free from errors: A survey of the errors in Briggs' Tables of Logarithms

has been carried out by A. J. Thompson in his *Logarithmica Britannica* (C.U.P. 1952), pages lxxix - lxxxiii.

[5] By this Briggs means Logarithms with a finite decimal expansion, such as 0.5, the square root of 10, here written as _.10. We need to be aware of the method used by Briggs regarding square root signs: Briggs prefixes 10 with _., being short for 'latus', or side. For geometrically, the number such as 10 is the area of a square with the side of this length. Although the reader may consider this as being a nuisance to use, rather than the familiar √ sign, used by Vlacq in the 2nd Edition of the *Arithmetica* (1627) – the symbol coming from Rudolph originally, and it had also been used by Thomas Digges in his *Pantometria* (1591), a book with which Briggs was quite familiar – it is in keeping with preserving the original notations of Briggs to retain it here. We will later have to contend with the methods used to show variables, brackets, etc. In Table 3 - 1, _.(6) 10 means $10^{1/6}$, etc.

§3.4. Caput III. [p.3.]

Unitatis Logarithmo constituto, proximum est ut alium quaeramus numerum, cuius usus est frequentissimus & maxime necessarius, eique tribuamus Logarithmum aliquem commodum, qui facillime & describi quoties opus fuerit & memoria teneri poterit. Ex omnibus autem numeris, nullis videtur Denario praestantior, aut huic negotio accommodatur. cuius Logarithmus esto 1,00000,00000,0000. Sunto igitur numeri praecipui Unitas & Denarius, eorumque Logarithmi 0, & 1,00000,00000,0000. Primos autem hosce quatuor statuimus, non necessitate aliqua adducti, sed pro arbitrio; nec operationum Arithmeticarum certitudinem spectantes (quae alijs modis plurimis ijsque 4.] diversissimis haberi poterit) sed facilitatem. Reliqui Logarithmi (cum in ijs constituendis non sit integrum quicquam agere pro arbitrio) sunt omnes hisce primis ita aptandi ut summo consensu a principio ad finem usque, ijsdem legibus obtemperent, eundemque quoties usus postulat effectum dent.

Horum autem quos quaerimus Logarithmorum alij sunt rationales, qui accurate inveniri & exhiberi poterunt, alij irrationales, quos non unde quaque perfectos, sed perfectis verisque proximos dabimus, ut vix tota unitas in numeris adeo magnis eorum cuiquam vel desit vel supersit. Rationales Logarithmi non istis tantum duobus debentur Unitati & *nales rithmi* Denario, quibus reliquorum omnium structura quasi fundamentis incumbit, sed omnino omnibus, qui quovis modo cum ijsdem in eandem continue proportionalium seriem conijci poterunt. ut hic vides:

A	*B*	*A*	*C*	*B*
1	0	1	1	0
10	100000	__. 10 - -- -- -	177827941	025000
100	200000	_. 10	316227766	050000
1000	300000	__.1000 - - - - -	562341325	075000
10000	400000	10	10	100000
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		_.1000	316227766	150000
		__.10000000	562341325	175000
		100 - - - -	100 - - - -	200000

A	*C*	*B*
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_.(2) 10	3162776017	0500
_.(3) 100	464158883360	0666 $\frac{2}{3}$
_.(6) 100000	681292069054	083 $\frac{1}{3}$
10	10 - - - - - -	10000

A sunt numeri ab Unitate continue proportionales. C valores eorundem in numeris absolutis, qui sunt etiam continue proporetionales. B eorum Logarithmi rationales & veri.

Reliqui omnes numeri qui in huiusmodi aliquam continue proportionalium serium (in qua dati duo numeri Unitas & Denartius siti sunt) cadere non possunt, habent Logarithmos irrationales, qui neque in numeris integri neque in partibus quas fractiones appelant accurate exprimi poterunt. Licet autem eos accuratos habere non possimus, poterimus tamen eos invenire adeo veris propinquos, ut si usum spectemus inter rationales & irrationales nihil intersit. Hoc ipso morbo Ptolomaei Canon Subtensarum, & subsequentum omnium ad haec usque tempora Astronomorum Canones Sinuum, Tangentium, & Secantium, sine aliquo graviore incommodo laborarunt.