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(*b*. Halle, Germany, 14 February 1839; *d*. Schramberg, near Tübingen, Germany. 29 August 1873)

mathematics, history of mathematics.

Hankel's father, the physicist Wilhelm Gottlieb Hankel was associate professor at Halle from 1847 and full professor at Leipzig from 1849. Hankel studied at the Nicolai Gymnasium in Leipzig, where he improved his Greek by reading the ancient mathematicians in the original. Entering Leipzig University in 1857, he studied with Moritz Drobisch, A. F. Moebius, Wilhelm Scheibner, and his father. In 1860 Hankel proceeded to Göttingen, where from Georg Riemann he acquired his special interest in the theory of functions. At this time he published his prizewinning *Zur allgemeinen Theorie der Bewegung der Flüssigkeiten* (Göttingen). The following year he studied in Berlin with Karl Weierstrass and Leopold Kronecker, and in 1862 he received his doctorate at Leipzig for *Ueber eine besondere Classe der symmetrischen Determinanten* (Göttingen, 1861). He qualified for teaching in 1863 and in the spring of 1867 was named associate professor at Leipzig. In the fall of that year he became full professor at Erlangen, where he married Marie Dippe. Called to Tübingen in 1869, he spent the last four years of his life there.

Hankel's contributions to mathematics were concentrated in three areas: the study of complex and higher complex numbers, the theory of functions, and the history of mathematics. His most important contribution in the first area was *Theorie der Complexen Zahlensysteme* (Leipzig, 1867), to which he had hoped to add a treatise on the functions of a complex variable. This work constitutes a lengthy presentation of much of what was then known of the real, complex, and hypercomplex number systems. In it Hankel presented algebra as a deductive science treating entities which are intellectual constructs. Beginning with a revised statement of George Peacock's principle of the permanence of formal laws, he developed complex numbers as well as such higher algebraic systems as Moebius' barycentric calculus, some of Hermann Grassmann's algebras, and W. R. Hamilton's quaternions. Hankel was the first to recognize the significance of Grassmann's long-neglected writings and was strongly influenced by them. The high point of the book lies in the section (pp. 106–108) in which he proved that no hypercomplex number system can satisfy all the laws of ordinary arithmetic.

In the theory of functions Hankel's major contributions were *Untersuchungen über die unendlich oft oscillirenden und unstetigen functionen* (Tübingen, 1870) and his 1871 article "Grenze" for the Ersch-Gruber *Encyklopädie*. In the former, he reformulated Riemann's criterion for integrability, placing the emphasis upon measure-theoretic properties of sets of points. After making explicit that functions do not possess general properties, he attempted a fourfold classification of functions, discussed the integrability of each type, and presented a method, based on his principle of the condensation of singularities, for constructing functions with singularities at every rational point. Although he confounded the notions of sets of zero content and nowhere-dense sets, his work marked an important advance toward modern integration theory. In "Grenze" he pointed out for the first time the importance of Bernard Bolzano's work on infinite series and published an example of a continuous function that was nondifferentiable at an infinite number of points. In a series of papers in *Mathematische Annalen*, Hankel showed the significance of what are now known as "Hankel functions" or "Bessel functions of the third kind."

Among Hankel's historical writings the best-known are his short *Entwicklung der Mathematik in den letzten Jahrhunderten* (Tübingen, 1869) and his long *Zur Geschichte der Mathematik in Alterthum und Mittelalter* (Leipzig, 1875). Although Moritz Cantor pointed out many errors in the latter book he, G. J. Allman Florian Cajori, T. L. Heath, and J. T. Merz have recognized the brilliance of Hankel's historical insight.

BIBLIOGRAPHY

I. Orignal Works A list of all Hankel's publications through 1875 will be found in *Bullettino di bibliographia edi storia delle scienze matematiche e fisiche*, **9** (1876), 297–308. This is completed by the following additions: *Untersuchungen über die unendlich oft oscillirenden und unstetigen Functionen*, republished in *Mathematische Annalen*, **20** (1882), 63–112, and as Ostwalds Klassiker der Exacten Wissenschaften, no. 153 (Leipzig, 1905), with comments by P. E. B. Jourdain. Also republished were *Entwicklung der Mathematik* (Tübingen, 1884) and, recently, *Zur Geschichte der Mathemzatik* (Hildesheim, 1965), with a foreword by J. E. Hofmann.

II. Secondary Literature. Hankel's life was discussed by W. von Zahn in *Mathematische Annalen*, **7** (1874), 583–590; and by M. Cantor in *Allgemeinen deutsche Biographie*, X (Leipzig, 1879), 516–519. For his work on complex numbers, see M. J. Crowe, *A History of Vector Analysis* (Notre Dame, Ind., 1967). His contributions to analysis are discussed in P. E. B. Jourdain, "The Development of the Theory of Transfinite Numbers," in *Archiv der Mathematik und Physik*, 3rd ser., **10** (1906), 254–281; and in Thomas Hawkins, *Lebesgue's Theory of Integration: Its Origins and Development* (Madison, Wis., 1970). J. E.

Hofmann's foreword to his republication of Hankel's *Zur Geschichte der Mathematik in Altertum und Mittelalter* (Hildesheim, 1965) contains a brief discussion of the quality of Hankel's historical writing as well as a portrait.

Michael J. Crowe