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(*fl*. China, *Ca*. 1261-1275)

mathematics.

The thirteenth century was perhaps the most significant period in the histroy of Chinese mathematics. It began with the appearance of Ch'in Chinu–shao's *Shu–shu chiu–change* in 1247, and the following year Li Chih issued an equally important work, the *Ts'eyvan hai–ching* These two great algebraistss were alater joined by Yang Hui (liternay surpassed those of his predecessors, and of whom wwe have absolutely no knowledge.The golden age of Chinese mathematics came to an end after the appearance of Chu Syih–chieh' *S su–yüan yü–chien* in 1303. Of the works of these four great Chinese mathematicians, those by Yang Hui have, until very recently, been the least studied and analyzed.

Nothing is known about the life of Yang Hui, except that he produced mathematical writings. From the prefaces to his works we learn that he was a native of Ch'ien–t'ang (now Hangchow). He seems to have been a civil servant, havin g served in T'ai–chou, and he had probably visited Su–chou (modern Soochow). His friends and acquaintances included Ch'en Chi hsien, Liu Pi–chien, Ch'iu Hsii–chii, and Shih Chung–yung, the last having colaborated with him on one of his works; but we know nothing else abot their personal history. Yang Hui also names as his teacher another mathematician, Liu 1, a native Chung–shan, of whom nothing is known.

In 1261 Yang Hui wrote *Hsiang–chieh chiuchang suan–fa* ("Detailed Analysis of the Mathematical Rules in the Nine Chapters"), a commentary on the old Chinese mathematical classic *Chiu–chang suan–shrr,* by Liu Hui. The present version of the *Hsiang–chief: chiu–chang suan–fa,* which is based on the edition in the *I–chia–t'ang ts' ungshu* (1842) collection, is incomplete and consists of only five chapters; two additional chapters have been restored from the *Yung–lo ta–tien* encyclopedia. Besides the original nine chapters of the *Chiu chang suan–shu,* Yang Hui's *Hsiang–chieh chiuchang suan–fa* included three additional chapters, making a total of twelve. According tot he preace written by Yang Hui, he had seselected 80 of the 246 problems in the *Chirr–chang suan–shit* for detailed discussion. The now–lost introductory chapter of the *Hsiang–chieh chia–chang sucur–fa* as we learn from the *shao–kuang*("Diminishing Breadth") chapter of the text and from a quotation in another of Yang Hui's works, *Suan–fa t'ung–pien pen–mo,* contained diagrams and illustrations.

Chapter 1, according to what Yang Hui describes in *Suan–fa t'ung–pien pen–mo* and *Ch'engch'su t'ung–pien suan–pao* dealt with the ordinary methods of multiplication and division. This chapter is also lost, but two of its problems have been restored from the *Yung–lo ta–tien* encyclopedia by Li Yen; it was entitled *Chu–chia suan–fa* Chapter 2, *Fang–t'ien* ("Surveying of Land") is now lost. Chapter 3, *su–mi*("Millet and Rice") is also lost, but three problems have been restored from the *Yung–lo ta–tien* encyclopedia. Chapter 4, *Ts' ui fen* ("Distribution by Progression"), is no longer extant; but eleven of its problems have been restored from the *Yung–lo ta–tien* encyclopedia. Chapter 5, *Shao–kuang* ("Diminishing Breadth"), also has been partially restored from the *Yang–lo tatien* encyclopedia. As for Chapter 6, *Shang–kung* ("Consultations on Engineering Works"), the *I chia–t' ts' ung–shu* collection contains thirteen problems (fifteen problems are missing). Chapter 7, *Chvn–shu* ("Impartial Taxation"); chapter 8, *Yingpu–tsu* ("Excess and Deficiency");chapter 9, Fang-ch'eng ("Calculation by Tabulation"), chapter 10, *Kou–ku* ("Right Angles"); and chapter 11, Tsuan-lei ("Reclassifications" remain more or less intact in the *I–chia–t ang ts'ung–shu* collection, except for one missing problem in chapter 7 and four in chapter 9.

In 1450 the Ming mathematician Wu ching wrote the *Chiu–chang hsiang–chu pi–lei suan–fa* ("Comparative Detailed Analysis of the Mathematical Rules in the Nine Chapters"), in which the chieh has shwon that Wu Ching's "old questions" were based on yang Hui's *Hsiang–chieh chiuchang suan–fa,* and he has been engaged in restoring this text. A substantial part of the *I–chia–t ang ts' ung–shu* and he was been engaged in restoring this text. A substantial part of the *i–chia–t'ang ts' ung–shu* edition of the text has been rendered into English by Lam Lay Youg of the University of Singapore.

Yang Hui published his second mathematical work, the two–volume *Jih–yung suan–fa* ("Mathematical Rules in Common Use"), in 1262. This book is no longer extant. Some sections have, however, been restored by Li Yen from the *chia suan–fa* in the *Yujng–lo ta–tien* encyclopedia. The book seems to be quite elementary.

In 1274 Yang Hui produced the *Ch'eng–ch'u t' ung–pien pen–mo*("Fundamental Mutual Changes in Multiplications and Divisions") in three volumes. The first volume was originally known as the *Suan–fa t'ung–pien pen–mo* ("Fundamental Mutual Changes in Calculations") the second as *Ch'eng–ch'u t'ung–pien suan–pao* "Fundamental Mutal Changes in Calculations" the second as *Ch'eng–ch'u t' ung–pien suan–pao* ("Treasure of Mathematical Arts on the Mutual Changes in Multiplications and Divisions") and the last volume, written in collaboration with Shih Chung–yung, was originally called *Fa–suan ch'v–yung pen–mo* ("Fundamentals of the Applications of Mathematics"). The next year Yank Hui wrote the *T ien mou* *pi–lei ch'eng–ch'u chieh–fa* ("Paractical Rules of Mathematics for Surveying") in two volumes. This was followed in the same year by the *h sii–ku chai–ch'i suan–fa* ("Continution of Ancient Mathematical Methods for Elucidating th eStrang Properties of numbers"), written after Yang Hui had been shown old mathematical documents by his friends Liu Pi–chien and Ch'iu Hsii–chii. Subsequently all seven volumes that Yang Hui wrote in 1274-1275 came to be known under a single title, *yang Hui suan–fa* ("The Mathematical Arts of Yang Hui"). The work was first printed in 1378 by the Ch'in–te shu–t'ang Press and was reprinted in Korea in 1433. A handwritten copy of the Korean reprint was made by the seventeenth–century Japanese mathematician Seki Takakazu. (A copy of the Korean reprint is in the Peking National Library.) Li Yen had Seki Takakazu's handwritten copy of the *Yang Hui suan–fa* dated 1661: it be came the property of the Academia Sinica after his death in 1963. At the beginning of the seventeenth century Mao Chin (1598-1652) made a handwriten copy of the fourteenth–century edition of the *Yang Hui suan–fa*.

All of Yang Hui's writings, including the *Hui suan–fa* seem to have been forgotten during the eighteenth century. Efforts were made in 1810 to reconstrut the text from the *Yung–lo ta–tien* encyclopedia by Juan Yuan (1764-1849), but they were confined to a portion of the *Hsü–ku chai–chai–ch'i suan–fa* in his *Chih–pu–tsu ts'ung–shu* collection that may have come from the restoration by Juan yuan. In 1814 Huang P'ei–lieh discovered an incomplete and disarranged copy of the *Yang Hui suan–fa* consisting of only six volumes, was incorported into the *I–chia–t'ang ts'ung–shu* collection by Yu Sung–nien (1842). Later reproduction of the text in the *T's ung–shu chi–ch'eng* collection (1936) is based on the version in the *I–chia–t'ang ts'ung–shu* collection. Some of the textual errors in the book have been corrected by Sung Ching–ch'ang in his *Yang Hui suan–fa chachi* A full English translation and commentary of the *Yang Hui suan–fa* was made by Lam Lay Yong in 1966 for her doctoral dissertation at the University of Singapore.

The *Hsiang–chieh chiu–change suan–fa* is perhaps the best–known, but certainly not the most interesting, of Yang Hui's writings In it he explains the questions and problems in the *Chiu–chang suanshu,* sometimes illustrating them with diagrams, and gives the detailed solutions. Problems of the same nature also are compared with eacy other. In the last chapter, the *T suan lei,* Yang Hui reclassifies all the 246 problems in the *Chiu–chang suanshu* in order of progressive difficulty, for the benefit of students of mathematics. Some examples of algbraic series given by Yang Hui in this book are

The portions restored from the *Yung–lo ta–tien* encyclopedia contain the earliest illustration of the "Pascal triangle." Yang Hui states that this diagram was derived from an earlier mathematical text, the *Shih–so suan–shu* Chia Hsien (fl. ca. 1050). This diagram shows the coefficients of the expansion of $(x+a)^n$ up to the sixth power. Another diagram showing coefficients up to the eighth power was later found in the early fourteenth–century work *Ssu–yiian yü–cheien* of Chu Shih–chieh. Other Chinese mathematicians using the Pascal triangle before Blaise Pascal were Wu Ching (1450), Chou Shu–hsüeh(1588),and Ch'eng Tawei(1592).

The *Tsuan–lei* also quotes a method of solving numerical equations higher than he second degree taken from Chia Hsien's *Shih–so suan–shu* The method is similar to that rediscovered independently in the early nineteenth century, by Ruffini and Horner, for solving numerial equations of all orders by continuous approximation. A method called the *tseng–ch' eng k'ai–li–fang fa* for solving a cubic equation x_3–1860867=0 is given in detail below.

The number 1860867 is set up in the second row of a counting board in which five rows are used–the top row *(shang)* is for the constant, the third row *(fang)* is for the coefficient of x, and the last row *(lien)* is for th ecoefficient of x^2, and the fourth row *(hsia–fa)* is for the coefficient of x^3. Thus 1 is placed in the last row, and this coefficient is shifted to the left, moving two place values at a time until it comes in line with the number 1860867, at the extreme left in this case, as shown in Figure la.

Since x lies between 100 and 200, 1 is placed at the hundreds' place of the first row. Multiplying this number by the number in the last row yields 1, which is entered on the fourth row as the *lien* Again, multiplying the number 1 on the top row by the *lien* gives 1, which is entered in the third row as the *fang* The number in the *fang* row is subtracted from the number in the same column in the *shih* row. The result is shown in Figure 1 *b*. The number on the last row is multiplied a second time by the number in the top row and the product is added to the *lien* The number in the *lien*is multiplied by eh number on the top row and added to the *fang* The number on the last row is multiplied a third time by the number on the top row, and the product is added to the number in the fourth row The result is shown in Figure l *c*.

The number in the third row *(fang)* is moved to the right by one place, that in the fourth row *(lien)* is moved to the right by two places, and that in the fifth row (lzsia–fa) is moved to the right by three places, as in Figure 1 *d*.

For the next approximation, x is found to lie between 120 and 130. Hence 2 is placed in the upper row in the tens' place. the same process is repeated, using 2 as the multiplier. We have 2x1 = 2, which is then added to the lien, giving a sum 32; and 2x 32 = 64, which, when added to the *fang* gives 364. Then 364 is multiplied by 2, giving 728, which is subtracted from 860867 to give 132867, as shown in Figure 1 *e*.

Then the *hsia–fa* is multiplied a second time by 2 and added to the *lien,* giving a sum of 34, which again is multiplied by 2and added to the *fang* giving 432.The *hsia–fa* is multiplied a third time by 2 and added to the number 34 in the *lien* row to give 36, as shown in Figure 1 *f*. The number in the *fang* row is now shifted one place, that in the *lien* row two palces, and that in the *hsia–fa* row three places to the right, as shown in Figure 1 *g*.

The last digit of x is found to be 3. This is placed on the top row in the units' column. Three times the *hsia–fa* gives 3, which, when added to the *lien* gives 363. 3x363= 1089, which when added to the *fang* gives 44289 Then 3 x 44289 =132867, which, when subtracted from the *shih* row, leaves no remainder, as shown in Figure 1h. The root is therefore 123.

The *Hsiang–chieh chiu–chang suan–fa* also contains a method for solving quartic equations called the *tseng san–ch'eng k'ai–fang fa* This involves the equation x^4 - 1,336,336 = 0. The method used is similar to that employed above for cubic equations. The solution is presented below in a slightly modified form, in order to show the resemblance to Horner's method.

Both the few remaining problems of the *Jih–yung suan–fa* restored from the *Yung–lo tatien* encyclo pedia and its title *Jih–yung* meaning "daily or common use," suggest that the book must be of an elementary and practical nature, although we no longer have access to its to its entire text. Two examples follow.

a. A certain article actually weighs 112 pounds How much does it weigh on the provincial steel-yard? Answer: 140 pounds. (Note that on the provincial steelyard 100 pounds would read 125 pounds.)

b. The weight of a certain article reads 391 pounds, 4 ounces on a provincial steelyard. What is its actual weight? Answer: 313 pounds.

The first Volume of the *Ch'en–ch'u t'ung–pien pen–mo (the Suan–fa t'ung–pien pern–mo)* gives a syllabus or program of study for the beginner that is followed by a detailed explanation of variations in the methods of multiplication of variations in the methods of multiplication. in it yang Hui shows how division can be conveniently replaced by multiplication by using the reciprocal of the divisor as the multiplier. For example, 2746 ÷ 25 = 27.46×4; 2746 ÷ 14.285 = 27.46×7; and 2746 ÷ 12.5 = 27.46×8. Sometimes he multiplies succesively by the factors of he multiplier–for example, 274 × 48= 274 × 6 × 8–and at other times he shows that the multiplier can be multiplied by the multiplicand–for example, 247 × 7360= 7360 × 247. Of special interest are the "additive" and "subtractive" methods that are the applied to multiplication. these methods are quite conveniently used on the counting board, or even on an abacus, where the numbers are set up rather than written on a piece of paper.If the multiplier is 21,31,41, 51, 61, 71, 81, or 91, multiplication can be performed by multiplying only with the tens' digit and the result, shifted one deciml place to the left, is added to the multiplicand. yang Hui gives a nuimber of examples to illustrate this method, such as 232 × 31 = 232 × 30 + 232; 234 × 410 = 234 × 400 + 2340. In the "subtractive" method of multiplication, if the multiplier x is a number of n digits, the multiplicand p is first multiplied by 10^n and from the result the product of the multiplicand and the difference between 10n and the multiplier x is subtracted, that is, $xp = 10^n$ p– (10^n– x)p. Yang Hui gives the example 26410x 7 = 264100–(26641x 3) = 1848700. The book ends with an account of how division can be performed.

In the second volume of the *Ch'eng–ch'u t'ungpien pen–mo (the Ch'eng–ch'u ung–pien suanpao)* Yang Hui proceeds further in showing how division can be avoided by multiplying with the reciprocal of the divisor. he also elaborates on the "additive" and "subtractiv" methods for multiplication. He states the rule that in division, the result remains unchanged if both the divident and the divisor are mnultiplied or divided by the same quantity. The examples he gives include the following.

The "subtractive" method for division is applied to cases *(e)* and *(f)* The steps for solving *(e)*are shown below.

yang Hui also states the rule that in multiplication, the result remains unchanged if the multiplicand is multiplied by a number and the multiplier by the reciprocal of the same number. Then he shows how to apply the rule to make the methods of "additive" and "subtractive" multiplication applicable. For example, 237 × 56 = × (56 × 2) = 118.5 × 112 = 11850 + 1185 + (118.5 × 2). The last part of the volume contains the division tables, the first instance of such tables in Chinese mathematical texts. They were later used by the Chinese in division operations involving the abacus.

The last volume of the *Ch'eng–ch'u t'ung–pien pen–mo (the Fa–suan ch'ü'yung – peris pen–mo)* gives various rapid methods for multiplication and division for multiplers and divisors from 2 to 300 that are based on the rules described in the first two volumes. For example, when the multiplier is 228, Yang Hui and his collaborator Shih Chung–yung recommend the useof the factors 12 and 19 and the successive application of the "additive" method of multiplication: and when the multiplier is 125, they recommend shifting the multiplicand three places to the left and then halving it three times successively.

The *T'ien–mou pilei ch'eng–ch'a chieh fa*, interesting mainly for its theory of equations, consists of two chapters. The first begins with a method for finding the area of a rectangular farm that is extended to problems involving other measures–weights, lengths, volumes, and money. These problems indicate that the length measurements for the sides of a rectangle can be employed as "dummy variables." Yang Hui was hence on the path leading to algebra, although neither he nor his Chinese contemporaries made extensive use of symbols. The text shows that Yang Hui had a highly developed conception of decimal places, simplified certain divisions by multiplication with reciprocals, and avoided the use of common fractions and showed his preference for decimal fractions. The words *chieh fa* (literally, "shorter method") in the title must have referred to these and other simplified methods that he introduced. Three different values for the ratio of the circumference to the diameter of a circle are used:3, 22/7, and 3.14.The rest of the first chapter deals with the area of the annulus, the isosceles triangle, and the trapezium; series; and arithmetic progressions exemplified by problems involving bundles of arrows with either square or circular cross sections.

The second chapter of the *T'ien–mom pi–lei ch'eng–ch'u chieh–fa* contains the earliest explanations of the Chinese methods for solving quadratic equations. For equations of the type $x^2 + 12x = 864$, Yang Hui recommends the *tai tsung k'ai fang* method, literally the method of extracting the root by attaching a side rectangle (*tsung*).The constant 864 is called *chi* ("total area"). If $x = 10x_1, + x_2$, $10x_1$ is called the ch'it shang ("first deliberation") and x^2, the tz'u shang ("second deliberation"), then $(10x_1,)^2$ is the fwng fti, x_2^2 the yü, and $l0x_1x_2$ the lien. Also, $12x = 12(10x_1 + x_2)$, with 12 $(10x_1)$ being called tsnng fong. and $12x_2$ the tsung.Five rows on the counting board are used: shang, chip, fang – fw, tsungfong, and yü or yü suan, in descending order. The constant 864 is first placed in the second row *(chih)*, then the coefficient of x on the fourth row *(tsang fang)*, and the coefficient for x^2 on the last row (yü scan). The coefficient of.r is moved one place to the left and that of xx 2 two places to the left, as shown in Figure 2a.The value of x lies between 20 and 30.The number 20, called the ch'it shang, is placed on the top row. Taking the number 2 of the ch'it sham, as the multiplier, the product with the vii sawn is 20.This is entered in the third row (fang–fn), as shown in Figure 2h. The number 2 of the ch'it shang is again used as the multiplier to find the products of the fang–fa and the tsmag–fang.The sum of these two products (640) is subtracted from the chih, giving a remainder 224. The third row, now known as lien, and the fourth row, now known as Wing, are moved one place to the right, while the vu scan is moved by two places, as shown in Figure 2*c*.

The "second deliberation" (tz'u shang) is found to be 4.This is placed in the first row after the number 2. The product of the "second deliberation" and the vii sawn, called vii, is added to the third row, which then becomes known as the lien vii (44 in this case). See Figure 2d.The sum of the products of the "second deliberation" and each of the lien vii and the tsmig (224), when subtracted from the chin, leaves no remainder. Hence x = 24.

The solution is also illustrated by Yang Hui in a diagram as shown in Figure 3.If $x = (10\ x_{,1} + x_2)$, where $x_1 = 2$ and $x_2 = 4$, then from the equation $(10x_1 + x_2)^2 + 12(10x_1 + x_2) = 864$ we obtain $100x_1^2 + 20x_1x_2 + x_2^2 + 120x_1 + 12x_2 = 864$. Here the fang–fw is given by $100x_1^2$, the two *lien* by $20\ x_1x_2$, yü by x_2^2, tsung fang by $120x_1$, tsung by $12x_2$,and the total chih by 864.

For equations of the type x^2 12x =864, Yang Hui gives two different methods: the *i chi k' ai fang* (extracting the root by increasing he area) and the *chien ts'ung k'ai fang* (extractiong the root by detaching a side rectangle, or *ts'ung)* For finding the smaller root of an equation of the type $-x^2 + 60x = 84$, Yang Hui recommends either of the two methods of *i yu* "adding a corner square" Finally, for finding the larger root of the same equation, he gives the *fan chi* "inverted area" method. The geometrical illustrations of these methods suggest that means for solving quadratic equations may have been first derived geometrically by Yang Hui. Negative roots are also discussed, and he general solutions given are similar to Horner's method.

In the second chapter of his *T ien–mou pilei ch' eng–ch'u chieh–fa* Yang Hui also descibes a method of solving equations of the type $x^4 = c$ as given by Liu I and Chia Hisien. There are also quadratic equations in which either the product and the difference of the which either the product and the difference of the two roots, or the product and the sum of the two roots re given, such as an equation in the form

$(x+y)^2=(x- y)^2+4xy$

In addition he gives a general formula for the positive roots in the form

The methods used by Yang Hui for solving quardratic equations appear to be more flexible than those used in the West. He also demonstrated how to solve a biquadratic equation of the form

$-5x^4+52x^2 + 128x^2=4096$

by means of the *san–ch'eng–fang* ("quartic root") method, which is very similar to the method rediscovered in the early nineteenth century by Horner and Ruffini. The rest of the chapter deals with dissection of areas: a rectangle cut off from a larger isosceles triangle, a trapezium cut way from a larger trapezium, and an annulus cut off from a larger annulus

The *Hsvü–ku chai–ch'i suan–fa* consists of two chapters. the first has recently aroused consider able interest because of its magic squares. It is found only in the rare Sung edition and is missing from the *I–chia–t ang ts' ung–shu* version, which is more commonly available. The *Hsü–ku chai–ch'i suan–fa* is the earliest Chinese text extant that gives magic squares higher than the third order an magic circles as well. In the preface Yang Hui disclaims originality in regard to its conhtents, saying that the material as among the old manuscipts and mathematical texts brought to him by his friends Liu Pi–chien and Ch'iu Hsii–chii. After Ch'en Tawei in his *Suan–fa t' ung–tsung*(1593), by Fang Chungt'ung in his *Shu tu yen* (1661), by Chang Ch'ao in his H sin–chai tsa tus (ca, 1670), and by Pao Chi'i–shou in his *Pi–nai–shan–fang chi*(ca. 1880). In 1935 Li Yen published a paper on Chinese magic sqarses adn reproduced the entire section on magic squares in Yang Hui's book, but with some misprints. In 1959 a subsection on magic squares and reproduced the entire section on magic squares and in Yang Hui's book, but with some misprints. In 1959 a subsection on magic squaeres was included in volume III of Joseph Needham's *Science and Civilisation in China* Through the courtesy of Li Yen, a microfilm of his personal copy of the Sung edition of *yang Hui suan–fa* containing the full chapter on magic squares, was obtained for the preparation of Lam Lay Yong's doctoral dissertation. Some of yang Hui's magic squares are shown in Figure 4a–4h.

Besides magic squares and magic circles, the first chapter deals with problems on indeterminate analysis, calendar computation, geometrical progressions, and volumes and areas of objects of various regular shapes. For indeterminate analysis

Yang Hui also gives the common names used in his time, such as *Ch'in Wang an tien ping* ("the Prince of Ch'in's secret method of counting soldiers"), *chien kuan shu* "method of cutting lengths of tube" (For futher details on the Chinese method of indeterminate analysis, see "Ch'in Chiushao.")

The first problem of the second chapter reads: "A number of pheasants and rabbits are placed together in the same cage. Thiry–five heads and ninety–four feet are counted. Find the number of each." Besides simultaneous linear equations of two unknowns, this chapter deals with three unknowns. The chapter then considers miscellaneous examples taken from several mathematical texts, including Liu Hui's *Chiu–chang suan–shu and Hai tao suan–ching, the Sun–tzu suan–ching, the Chang Ch'iu–chien suan–ching, the Ying–yung suan–fa* The last three mathematical texts were printed during the eleventh and twelfth centuries in China, but are now lost. It is only through the works of Yang Hui that fragments of them and some of the other texts printed in the same era are extant. In the last problem of the chapter, Yang Hui gives a detailed analysis of the method employed by Liu Hui in his *Hai–tao suan–ching* It is interesting that Yang Hui's first publication, the *Hsiang–chieh chiu–chang suan–fa* is a study of Liu Hui's *Chiu–chang suan–shu* which had been authoritative in China for about 1,000 years. Thus, with the last problem in his last publication, Yang Hui had completed a total analysis Liu Hui's writings

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